Pág. 335, R3

$$\int\sqrt{1-x^2}\,dx=\left\{\begin{array}{l}x=sen\ t\rightarrow t=arcsen\ x\\ dx=\cos t\ dt\end{array}\right\}=\int\sqrt{1-sen^2\ t}\ \cos t\ dt=\int\sqrt{\cos^2\ t}\ \cos t\ dt$$

$$=\int\cos t\cos t\ dt=\int\cos^2\ t\ \ dt=$$

$$\left.\begin{array}{l}1=sen^2x+\cos^2 x\\ \cos 2x=\cos^2 x-sen^2x\end{array}\right\}\rightarrow\ 1+\cos 2x=2\cos^2 x$$

$$\cos^2 x=\frac{1+\cos 2x}{2}=\frac{1}{2}+\frac{\cos 2x}{2}$$

$$=\int\left(\frac{1}{2}+\frac{\cos 2t}{2}\right)dt=\int\frac{1}{2}\,dt+\int\frac{\cos 2t}{2}\,dt=\frac{1}{2}t+\frac{1}{2}\int\cos 2t\ dt=$$

$$\int\cos 2t\ dt=\left\{\begin{array}{l}u=2t\\ du=2dt\ \ \rightarrow\ \ \dfrac{du}{2}=dt\end{array}\right\}=\int\cos u\ \frac{du}{2}=\frac{1}{2}\ sen\ u=\frac{1}{2}\ sen\ 2t$$

$$=\frac{1}{2}t+\frac{1}{2}\frac{1}{2}\ sen\ 2t+C=\frac{1}{2}t+\frac{1}{4}\ sen\ 2t+C=\frac{1}{2}\ arcsen\ x+\frac{1}{4}\ sen\left(2\ arcsen\ x\right)+C$$

5) $$\int\sqrt{4-x^2}\,dx=\left\{\begin{array}{l}x=2\ sen\ t\rightarrow\dfrac{x}{2}=sen\ t\\ dx=2\cos t\ dt\end{array}\right\}=\int\sqrt{4-(2sen\ t)^2}\ 2\cos t\ dt=$$

$$=\int\sqrt{4-4sen^2\ t}\ (2\cos t)\,dt=\int\sqrt{4\left(1-sen^2\ t\right)}\ (2\cos t)\,dt=\int 2\cos t\ (2\cos t)\,dt=$$

$= seguir$

Integración por partes. Pág. 336. $\int u\,dv = u\,v - \int v\,du$

$$\int x\,e^x dx = \left\{\begin{matrix} u = x & \rightarrow & du = dx \\ dv = e^x dx & \left[\rightarrow \int dv = \int e^x dx\right] \rightarrow & v = e^x \end{matrix}\right\} =$$

$$= \left[\int u\,dv = uv - \int v du\right] = x\,e^x - \int e^x\,dx = x\,e^x - e^x + C$$

$$\int x^3\,Ln\,x\,dx = \left\{\begin{matrix} u = Ln\,x & \rightarrow & du = \dfrac{dx}{x} \\ dv = x^3\,dx & \rightarrow & v = \dfrac{x^4}{4} \end{matrix}\right\} = \frac{x^4}{4}\,Ln\,x - \int \frac{x^4}{4}\,\frac{dx}{x} =$$

$$= \frac{x^4}{4}\,Ln\,x - \frac{1}{4}\int x^3 dx = \frac{x^4}{4}\,Ln\,x - \frac{1}{4}\,\frac{x^4}{4} + C = \frac{x^4}{4}\,Ln\,x - \frac{x^4}{16} + C$$

$$\int Ln\,x\,dx = \left\{\begin{matrix} u = Ln\,x & \rightarrow & du = \dfrac{dx}{x} \\ dv = dx & \rightarrow & v = x \end{matrix}\right\} = x\,Ln\,x - \int x\,\frac{dx}{x} = x\,Ln\,x - \int dx =$$

$$= x\,Ln\,x - x + C$$

$$1)\ \int x\,sen\,x\,dx = \left\{\begin{matrix} u = x & \rightarrow & du = dx \\ dv = sen\,x\,dx & \rightarrow & v = -\cos\,x \end{matrix}\right\} = -x\cos\,x - \int -\cos\,x\,dx =$$

$$= -x\cos\,x + \int \cos\,x\,dx = -x\cos\,x + sen\,x + C$$

$$2)\ \int x\,arctg\,x\,dx = \left\{\begin{matrix} u = arctg\,x & \rightarrow & du = \dfrac{dx}{1+x^2} \\ dv = x\,dx \rightarrow & v = \dfrac{x^2}{2} & \end{matrix}\right\} == \frac{x^2}{2} arctg\,x - \int \frac{x^2}{2}\,\frac{dx}{1+x^2} = (*)$$

$$\int \frac{x^2}{2}\,\frac{dx}{1+x^2} = \frac{1}{2}\int \frac{x^2}{1+x^2}dx = \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx = \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx =$$

$$= \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx = \frac{1}{2}(x - arctg\,x)$$

$$(*) = \frac{x^2}{2} arctg\,x - \frac{1}{2}(x - arctg\,x) + C$$