$$\int \frac{5x^2-6x+3}{x^3-3x^2+x-3}dx=$$

	1	-3	1	-3
3		3	0	3
	1	0	1	0

$$\frac{5x^2-6x+3}{x^3-3x^2+x-3}=\frac{5x^2-6x+3}{(x-3)\left(x^2+1\right)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+1}=\frac{A\left(x^2+1\right)+\left(Bx+C\right)(x-3)}{(x-3)\left(x^2+1\right)}$$

$$5x^2-6x+3=A\left(x^2+1\right)+\left(Bx+C\right)(x-3)$$

$x=3;\quad 30=10A;\quad A=3$

$x=0;\quad 3=A-3C;\quad 3=3-3C;\quad 0=-3C;\quad C=0$

$x=1;\quad 2=2A+(B+C)(-2);\quad 2=6+B(-2);\quad 2-6=-2B;\quad -4=-2B;\quad B=2$

$$\frac{5x^2-6x+3}{x^3-3x^2+x-3}=\frac{3}{x-3}+\frac{2x}{x^2+1}$$

$$\int \frac{5x^2-6x+3}{x^3-3x^2+x-3}dx=\int \frac{3}{x-3}dx+\int \frac{2x}{x^2+1}dx=3\ Ln\left|x-3\right|+Ln\left|x^2+1\right|+C$$

****

$$\int \frac{5x+1}{x^3+2x^2-x-2}\,dx =$$

	1	2	-1	-2
1		1	3	-2
	1	3	2	0
-1		-1	-2	
	1	2	0	
-2		-2		
	1	0		

$$\frac{5x+1}{x^3+2x^2-x-2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{A(x+1)(x+2)+B(x-1)(x+2)+C(x-1)(x+1)}{(x-1)(x+1)(x+2)}$$

$$5x+1 = A(x+1)(x+2)+B(x-1)(x+2)+C(x-1)(x+1)$$

$x=1;\quad 6=6A;\quad A=1$

$x=-1;\quad -4=B(-2);\quad B=2$

$x=-2;\quad -9=3C;\quad C=-3$

$$\frac{5x+1}{x^3+2x^2-x-2} = \frac{1}{x-1} + \frac{2}{x+1} - \frac{3}{x+2}$$

$$\int \frac{5x+1}{x^3+2x^2-x-2}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{2}{x+1}\,dx - \int \frac{3}{x+2}\,dx =$$
$$= Ln|x-1| + 2Ln|x+1| - 3Ln|x+2| + C$$

Pág. 344,
8)

$$\int \frac{x^4+2x^3+3x^2+3x+3}{x^3+2x^2+3x}\,dx$$

x^4	$+2x^3$	$+3x^2$	$+3x$	$+3$	x^3+2x^2+3x
$-x^4$	$-2x^3$	$-3x^2$			x
			$3x$	$+3$	

$$\frac{x^4+2x^3+3x^2+3x+3}{x^3+2x^2+3x} = x + \frac{3x+3}{x^3+2x^2+3x}$$

$$\int \frac{x^4+2x^3+3x^2+3x+3}{x^3+2x^2+3x}\,dx = \int x\,dx + \int \frac{3x+3}{x^3+2x^2+3x}\,dx = \frac{x^2}{2} + Ln|x| - \sqrt{2}\ arctg\left(\frac{x+1}{\sqrt{2}}\right) + C$$

Calculemos la segunda integral,

$$\int \frac{3x+3}{x^3+2x^2+3x}dx$$

Obtengamos las raíces del denominador:

	1	2	3	0
0		0	0	0
	1	2	3	0

$$x^2+2x+3=0; \quad x=\frac{-2\pm\sqrt{2^2-4\cdot 1\cdot 3}}{2\cdot 1} \ no\exists$$

$$x^3+2x^2+3x=x\left(x^2+2x+3\right)$$

$$\frac{3x+3}{x^3+2x^2+3x}=\frac{A}{x}+\frac{Bx+C}{x^2+2x+3}=\frac{A\left(x^2+2x+3\right)+\left(Bx+C\right)x}{x\left(x^2+2x+3\right)}$$

$$3x+3=A\left(x^2+2x+3\right)+\left(Bx+C\right)x$$

$$x=0; \quad 3=3A; \quad A=1$$

$$x=1; \quad 4=6\cdot 1+(B+C); \quad B+C=-2$$

$$x=-1; \quad 0=2+(-B+C)(-1); \quad 0=2+B-C; \quad B-C=-2$$

$$sumando: \quad 2B=-4; \quad B=-2$$

$$-2+C=-2; \quad C=0$$

Luego: $\dfrac{3x+3}{x^3+2x^2+3x}=\dfrac{1}{x}+\dfrac{-2x}{x^2+2x+3}$

Por lo que: $\displaystyle\int\frac{3x+3}{x^3+2x^2+3x}dx=\int\frac{1}{x}dx-\int\frac{2x}{x^2+2x+3}dx=Ln|x|-\sqrt{2}\ arctg\left(\frac{x+1}{\sqrt{2}}\right)$

Calculemos la 2ª integral,

$$\int\frac{2x}{x^2+2x+3}dx=\int\frac{2x+2-2}{x^2+2x+3}dx=\int\frac{2x+2}{x^2+2x+3}dx-\int\frac{2}{x^2+2x+3}dx$$

$$\int\frac{2}{x^2+2x+3}dx=\int\frac{2}{(x+1)^2+2}dx=\int\frac{\frac{2}{2}}{\frac{(x+1)^2+2}{2}}dx=\int\frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}dx=$$

$$x^2+2x+3=(x+1)^2+2$$

$$=\sqrt{2}\int\frac{\frac{1}{\sqrt{2}}}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}dx=\sqrt{2}\ arctg\left(\frac{x+1}{\sqrt{2}}\right)$$

Calcular las siguientes integrales:

$$\int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx =$$

$$\int \frac{2x^2 - 17}{x^3 - 3x - 2} dx =$$