

Calcular las siguientes integrales:

$$\int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx =$$

$$\begin{array}{r} x^4 \quad \quad \quad -2x \quad -6 \quad \left| \frac{x^3 + x^2 - 2x}{x-1} \right. \\ -x^4 \quad -x^3 \quad +2x^2 \\ \hline \quad -x^3 \quad +2x^2 \quad -2x \\ \quad +x^3 \quad +x^2 \quad -2x \\ \hline \quad \quad 3x^2 \quad -4x \quad -6 \end{array}$$

$$\frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} = x - 1 + \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x}$$

$$\int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx = \int x dx - \int 1 dx + \int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx$$

$\int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx$	$\begin{array}{r rr} 1 & 1 & -2 \\ & 1 & 2 \\ \hline & 1 & 2 & 0 \end{array}$
$x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x-1)(x+2)$	

$$\int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx$$

$$x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x-1)(x+2)$$

$$\frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2} = \frac{A(x-1)(x+2) + Bx(x+2) + Cx(x-1)}{x(x-1)(x+2)}$$

$$3x^2 - 4x - 6 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$$x=0; \quad -6 = -2A; \quad A=3$$

$$x=1; \quad -7 = 3B; \quad B = \frac{-7}{3}$$

$$x=-2; \quad 14 = 6C; \quad C = \frac{14}{6} = \frac{7}{3}$$

$$\int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx = \int \frac{3}{x} dx + \int \frac{-7}{x-1} dx + \int \frac{7}{x+2} dx = 3 \ln|x| - \frac{7}{3} \ln|x-1| + \frac{7}{3} \ln|x+2|$$

Finalmente,

$$\int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx = \int x dx - \int 1 dx + \int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx = \frac{x^2}{2} - x + 3 \ln|x| - \frac{7}{3} \ln|x-1| + \frac{7}{3} \ln|x+2| + C$$

$$\int \frac{2x^2 - 17}{x^3 - 3x - 2} dx =$$

$$\begin{array}{c|cccc} & 1 & 0 & -3 & -2 \\ -1 & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & 0 \\ -1 & & -1 & 2 & \\ \hline & 1 & -2 & & 0 \end{array}$$

$$x^3 - 3x - 2 = (x+1)^2 (x-2)$$

$$\frac{2x^2 - 17}{x^3 - 3x - 2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2} = \frac{A(x+1)(x-2) + B(x-2) + C(x+1)^2}{(x+1)^2 (x-2)}$$

$$2x^2 - 17 = A(x+1)(x-2) + B(x-2) + C(x+1)^2$$

$$x = -1; \quad -15 = -3B; \quad B = 5$$

$$x = 2; \quad -9 = 9C; \quad C = -1$$

$$x = 0; \quad -17 = -2A - 2B + C; \quad -17 = -2A - 10 - 1; \quad -17 + 11 = -2A; \quad -6 = -2A; \quad A = 3$$

$$\int \frac{2x^2 - 17}{x^3 - 3x - 2} dx = \int \frac{3}{x+1} dx + \int \frac{5}{(x+1)^2} dx + \int \frac{-1}{x-2} dx = 3 \ln|x+1| - \frac{5}{x+1} - \ln|x-2| + C$$

$$\int \frac{5}{(x+1)^2} dx = 5 \int (x+1)^{-2} dx = 5 \frac{(x+1)^{-2+1}}{-2+1} = -5(x+1)^{-1} = \frac{-5}{x+1}$$

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$$\int x^4 e^{x^5} dx = \left\{ t = x^5 \rightarrow dt = 5x^4 dx \rightarrow \frac{dt}{5} = x^4 dx \right\} =$$

$$= \int e^t \frac{dt}{5} = \frac{1}{5} \int e^t dt = \frac{1}{5} e^t + C = \frac{1}{5} e^{x^5} + C$$

$$\int x \operatorname{sen} x^2 dx = \left\{ t = x^2 \rightarrow dt = 2x dx \rightarrow \frac{dt}{2} = x dx \right\} =$$

$$= \int \operatorname{sen} t \frac{dt}{2} = \frac{1}{2} (-\cos t) + C = \frac{-\cos x^2}{2} + C$$

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