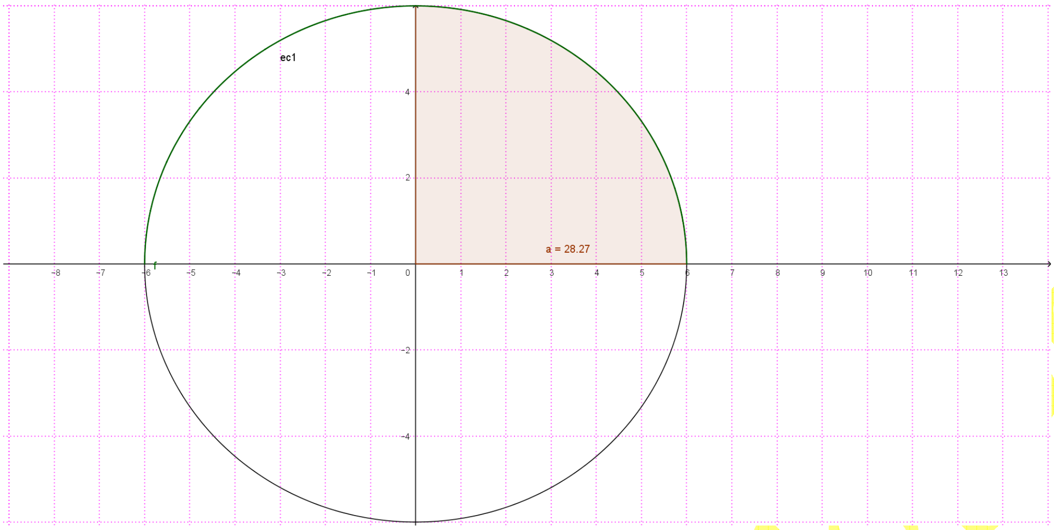
12c)

$$\int_0^6 \sqrt{36-x^2}\,dx = \frac{36\pi}{4} = 9\pi$$

$$y=\sqrt{36-x^2};\quad y^2=36-x^2;\quad x^2+y^2=36\quad circunferencia\ de\ radio\ 6\ y\ centro\ (0,0)$$

$$A_c = \pi\cdot 6^2 = 36\pi$$

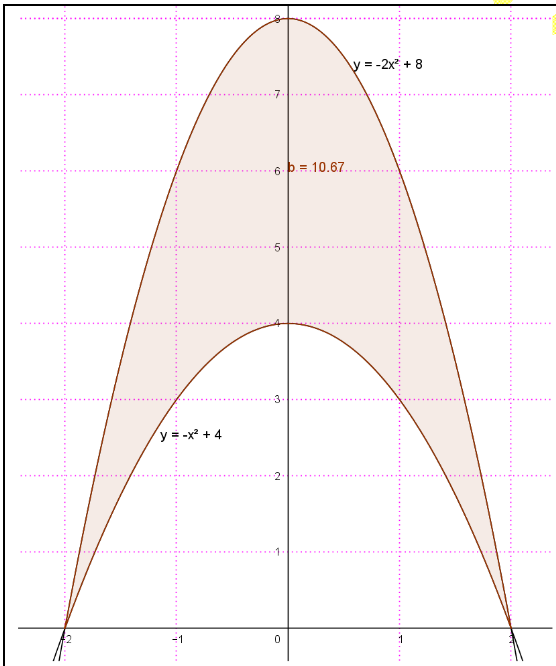


15)
a) Área entre las curvas $y = 4 - x^2$ e $y = 8 - 2x^2$
1º) Puntos de corte entre ellas.

$$4-x^2=8-2x^2;\quad 2x^2-x^2=8-4;\quad x^2=4;\quad x=\pm 2$$

$$De\quad y=4-x^2;\quad x=0,\quad y=4$$

$$De\quad y=8-2x^2;\quad x=0,\quad y=8$$



$$\int_{-2}^{2}\left[-2x^2+8-\left(4-x^2\right)\right]dx = \int_{-2}^{2}\left(-x^2+4\right)dx =$$

$$=\left[-\frac{x^3}{3}+4x\right]_{-2}^{2} = \left(-\frac{2^3}{3}+4\cdot 2\right)-\left(-\frac{(-2)^3}{3}+4(-2)\right)=$$

$$=\frac{16}{3}-\left(\frac{-16}{3}\right)=\frac{32}{3}$$

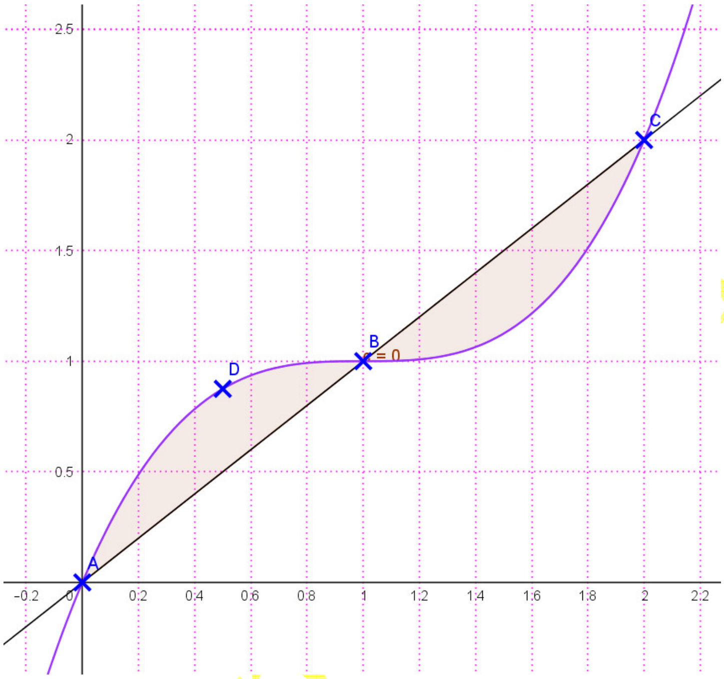
$$El\ área\ a\ calcular\ es\ \frac{32}{3}u^2$$

c) Área entre $y = x^3 - 3x^2 + 3x$ e $y = x$

Puntos de corte entre ellas.

$x^3 - 3x^2 + 3x = x; \quad x^3 - 3x^2 + 2x = 0;$

$$x\left(x^2 - 3x + 2\right) = 0 \begin{cases} x = 0 \\ x^2 - 3x + 2 = 0 \begin{cases} x = 1 \\ x = 2 \end{cases} \end{cases}$$



Los puntos de corte:

x	y
0	0
1	1
2	2

$y = x^3 - 3x^2 + 3x$:

x	y
-1	-7
0´5	$7/8 = 0´875$
1´5	$9/8 = 1´125$

$$\int_0^1 \left[x^3 - 3x^2 + 3x - x\right]dx = \int_0^1 \left[x^3 - 3x^2 + 2x\right]dx = \left[\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}\right]_0^1 =$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 = \left(\frac{1^4}{4} - 1^3 + 1^2\right) - \left(\frac{0^4}{4} - 0^3 + 0^2\right) = \frac{1}{4}$$

$$\int_1^2 \left[x - x^3 + 3x^2 - 3x\right]dx = \int_1^2 \left[-x^3 + 3x^2 - 2x\right]dx = \left[-\frac{x^4}{4} + 3\frac{x^3}{3} - 2\frac{x^2}{2}\right]_1^2 =$$

$$= \left[-\frac{x^4}{4} + x^3 - x^2\right]_1^2 = \left(-\frac{2^4}{4} + 2^3 - 2^2\right) - \left(-\frac{1^4}{4} + 1^3 - 1^2\right) = 0 + \frac{1}{4}$$

$$Área = \frac{1}{4} + \frac{1}{4} = 0´5\ u^2$$