Martes, 2 de junio

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Área limitada por

$y = x^3 - x \quad e \quad y = -x^2$

$Puntos\ de\ corte: \quad x^3 - x = -x^2; \quad x^3 - x + x^2 = 0; \quad x\left(x^2 + x - 1\right) = 0 \left\langle \begin{array}{l} x = 0 \\ x^2 + x - 1 = 0 \end{array} \right.$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{2} = \left\langle \begin{array}{l} x_1 = \dfrac{-1+\sqrt{5}}{2} = 0'61803 \\ x_3 = \dfrac{-1-\sqrt{5}}{2} = -1'61803 \end{array} \right.$$

$Puntos\ de\ corte: \quad x_1 = \dfrac{-1-\sqrt{5}}{2}, \quad x = 0 \quad y \quad x_3 = \dfrac{-1+\sqrt{5}}{2}$

$$\int_{x_1}^{0} \left(x^3 - x + x^2\right)dx = \left[\frac{x^4}{4} - \frac{x^2}{2} + \frac{x^3}{3}\right]_{x_1}^{0} = \left(\frac{0^4}{4} - \frac{0^2}{2} + \frac{0^3}{3}\right) - \left(\frac{{x_1}^4}{4} - \frac{{x_1}^2}{2} + \frac{{x_1}^3}{3}\right) = 0 - (-1'007514) = 1'007514$$

$$\int_{0}^{x_3} \left(x^3 - x + x^2\right)dx = \left[\frac{x^4}{4} - \frac{x^2}{2} + \frac{x^3}{3}\right]_{0}^{x_3} = \left(\frac{{x_3}^4}{4} - \frac{{x_3}^2}{2} + \frac{{x_3}^3}{3}\right) - \left(\frac{0^4}{4} - \frac{0^2}{2} + \frac{0^3}{3}\right) = -0'07582 - 0 = -0'07582$$

$Área = 0'07582 + 1'007514 = 1'083334\ u^2$

f)

$y = 2 - x^4 \quad e \quad y = x^2$

$Puntos\ de\ corte: \quad 2 - x^4 = x^2; \quad 2 - x^4 - x^2 = 0; \quad x^4 + x^2 - 2 = 0; \quad \begin{cases} z = x^2 \\ z^2 = x^4 \end{cases}$

$$x^2 = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm 3}{2} = \left\langle \begin{array}{l} {x^2}_1 = \dfrac{-1+3}{2} = 1 \quad \rightarrow \quad x = \pm\sqrt{1} = \left\langle \begin{array}{l} 1 \\ -1 \end{array} \right. \\ {x^2}_2 = \dfrac{-1-3}{2} = -2 \quad \rightarrow \quad x = \pm\sqrt{-2}\ no\ \exists \end{array} \right.$$

$Puntos\ de\ corte: \quad x_1 = -1 \quad y \quad x_2 = 1$

$$\int_{-1}^{1} \left(x^4 + x^2 - 2\right)dx = \left[\frac{x^5}{5} + \frac{x^3}{3} - 2x\right]_{-1}^{1} = \left(\frac{1^5}{5} + \frac{1^3}{3} - 2 \cdot 1\right) - \left(\frac{(-1)^5}{5} + \frac{(-1)^3}{3} - 2(-1)\right) = -\frac{22}{15} - \left(\frac{22}{15}\right) = \frac{-44}{15} =$$

$Área = \dfrac{44}{15}\ u^2$

Teorema del valor medio del cálculo integral.

f(x) continua en [a,b] entonces existe un c $\in$ (a,b) / $\int_a^b f(x)dx = f(c)\,(b-a)$

Teorema fundamental del cálculo integral

f(x) continua en [a,b] entonces $F(x) = \int_a^x f()\ dx$ es derivable y F´(x) = f(x)

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$F(x) = \int_0^x \cos t\ dt, \quad calcula \quad F'(x)$

Como f(t) = cos t es continua en $\Re$, podemos aplicar el TFCI,

F´(x) = cos x

$F(x) = \int_x^0 \cos t\ dt, \quad calcula \quad F'(x)$

$F(x) = \int_x^0 \cos t\ dt = -\int_0^x \cos t\ dt$

$F'(x) = -\cos x$