1) Representa $y=\dfrac{x^2+1}{x-1}$

$$y=\frac{x^2+1}{x-1}$$

a) $\mathrm{Dom}\ y = \mathbb{R} \sim \{1\}$

$x-1=0 \rightarrow x=1$

b) Ramas

<u>A.V</u>

$x=1$

$$\lim_{x\to 1} \frac{x^2+1}{x-1} = \frac{1^2+1}{1-1} = \frac{2}{0} = \infty \Rightarrow x=1 \text{ es a.v}$$

$$\lim_{x\to 1^-} \frac{x^2+1}{x-1} \underset{\downarrow}{\overset{x=0'9}{=}} \frac{+}{-}\infty = -\infty$$

$$\lim_{x\to 1^+} \frac{x^2+1}{x-1} \underset{\downarrow}{\overset{x=1'1}{=}} \frac{+}{+}\infty = +\infty$$

$x=1$

<u>A.H</u>

$$\lim_{x\to -\infty} \frac{x^2+1}{x-1} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to -\infty} \frac{x^2}{x} = \lim_{x\to -\infty} x = -\infty \Rightarrow \text{No hay a.h.}$$

A.O.

$$\begin{array}{l} grd(num) = 2 \\ -\ grd(den) = 1 \\ \hline \qquad\qquad 1 \Rightarrow \text{Hay A.O.} \end{array}$$

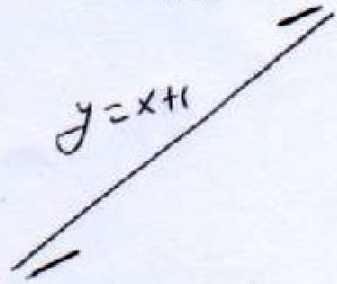
$$\begin{array}{r|l} x^2+1 & x-1 \\ \cline{2-2} -x^2+x & x+1 \\ \cline{1-1} x+1 & \\ -x+1 & \\ \cline{1-1} 2 & \end{array}$$

Luego $y = x+1$ es la A.O.

$$\frac{x^2+1}{x-1} = (x+1) + \frac{2}{x-1}$$

En $-\infty$, $x = -1000 \rightarrow \frac{2}{-1000-1} = \frac{2}{-1001} = -0'\ldots$

En $+\infty$, $x = 1000 \rightarrow \frac{2}{1000-1} = \frac{2}{999} = +0'\ldots$



x	$y=x+1$
0	1
-1	0

c) Ptos de corte

$x = 0 \rightarrow y = \frac{0^2+1}{0-1} = \frac{1}{-1} = -1 \rightarrow (0,-1)$

$y = 0 \rightarrow \frac{x^2+1}{x-1} = 0 \rightarrow x^2+1 = 0 \rightarrow x^2 = -1$ No tiene soluciones

d) $y' = \frac{2x(x-1) - (x^2+1)\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2-1}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$

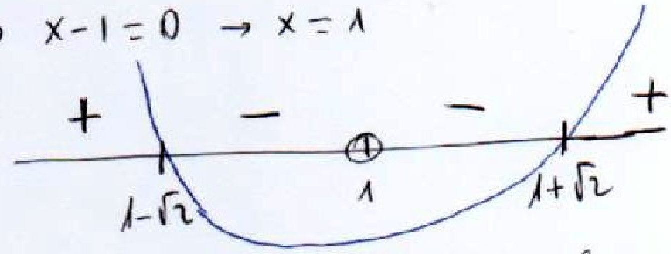
e) Monotonía

Signo de y'

$$x^2-2x-1=0 \rightarrow x=\frac{2\pm\sqrt{(-2)^2-4\cdot 1\cdot(-1)}}{2\cdot 1}=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm 2\sqrt{2}}{2}=$$

$$=1\pm\sqrt{2} \begin{cases} 1+\sqrt{2}\simeq 2'41 \\ 1-\sqrt{2}\simeq -0'41 \end{cases}$$

$$(x-1)^2=0 \rightarrow x-1=0 \rightarrow x=1$$



En y' el denominador está elevado al cuadrado, luego siempre será positivo. El signo de y' depende del numerador que es un polinomio de 2º grado con coeficiente de x^2 positivo y raíces $1-\sqrt{2}$ y $1+\sqrt{2}$

Creciente $(-\infty, 1-\sqrt{2}) \cup (1+\sqrt{2}, +\infty)$

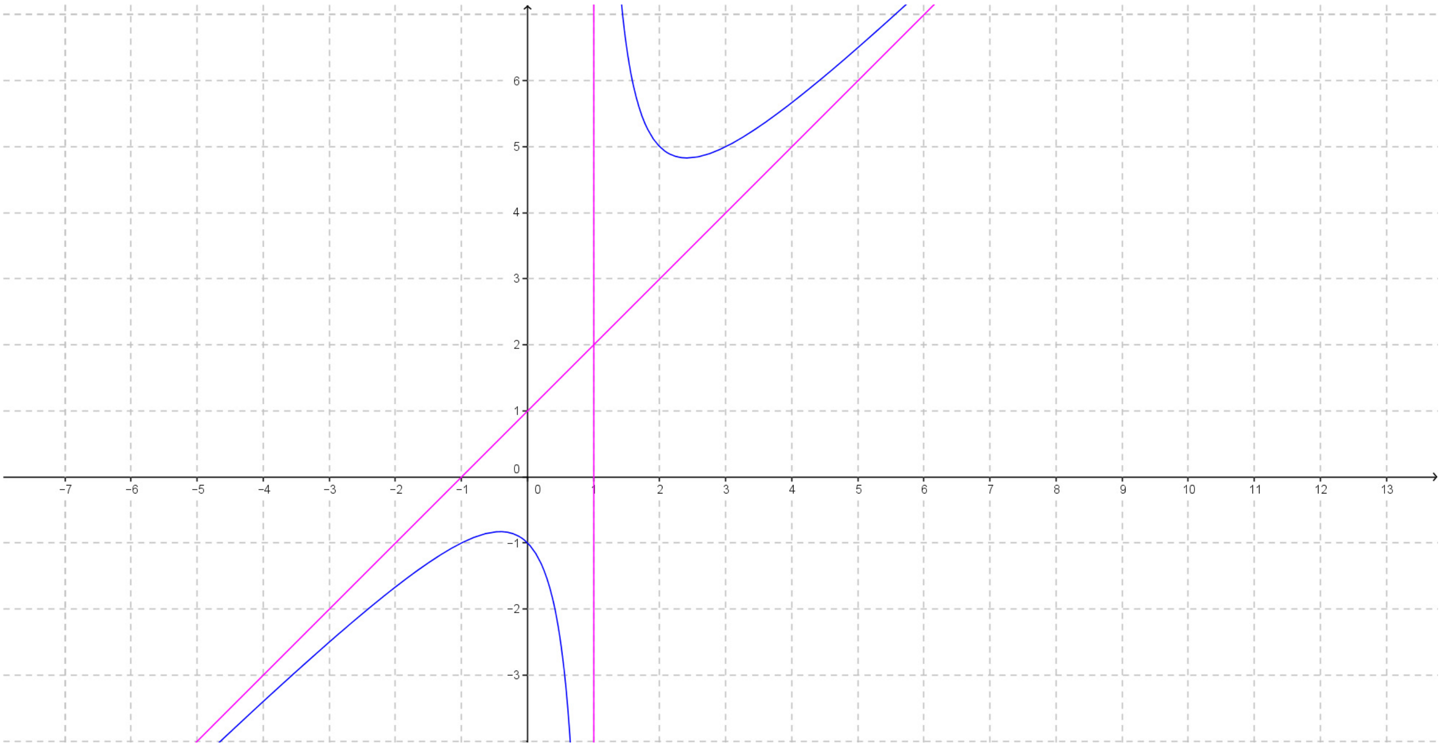
Decreciente $(1-\sqrt{2}, 1) \cup (1, 1+\sqrt{2})$

f) Máximo en $x=1-\sqrt{2} \rightarrow y=\frac{(1-\sqrt{2})^2+1}{1-\sqrt{2}-1}=\frac{4-2\sqrt{2}}{-\sqrt{2}}=2-2\sqrt{2}\simeq -0'83$

Mínimo en $x=1+\sqrt{2} \rightarrow y=\frac{(1+\sqrt{2})^2+1}{1+\sqrt{2}-1}=\frac{4+2\sqrt{2}}{\sqrt{2}}=2+2\sqrt{2}\simeq 4'83$

Máximo en $(1-\sqrt{2}, 2-2\sqrt{2}) \simeq (-0'41, -0'83)$

Mínimo en $(1+\sqrt{2}, 2+2\sqrt{2}) \simeq (2'41, 4'83)$



0
1
2
3
4
5
6
−1
−2
−3
−7
−6
−5
−4
7
8
9
10
11
12
13

2) Representa $y = \dfrac{x^2}{x^2 - 4x + 3}$, sabiendo que $y' = \dfrac{-4x^2 + 6x}{\left(x^2 - 4x + 3\right)^2}$.

a) $\text{Dom } y = \mathbb{R} \sim \{1, 3\}$

$x^2 - 4x + 3 = 0 \Rightarrow x = \dfrac{4 \pm \sqrt{(-4)^2 - 4.1.3}}{2.1} = \dfrac{4 \pm \sqrt{16-12}}{2} = \dfrac{4 \pm \sqrt{4}}{2} = \dfrac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

b) Ramas

AV

$x = 1$

$\lim\limits_{x \to 1} \dfrac{x^2}{x^2 - 4x + 3} = \dfrac{1}{1-4+3} = \dfrac{1}{0} = \infty \Rightarrow x = 1$ es a.v.

$\lim\limits_{x \to 1^-} \dfrac{x^2}{x^2-4x+3} \overset{x=0'9}{=} \dfrac{0'9^2}{0'9^2 - 4.0'9 + 3} \infty = \dfrac{+}{+} \infty = +\infty$

$\lim\limits_{x \to 1^+} \dfrac{x^2}{x^2-4x+3} \overset{x=1'1}{=} \dfrac{1'1^2}{1'1^2 - 4.1'1 + 3} \infty = \dfrac{+}{-} \infty = -\infty$

$x=1$

$x = 3$

$\lim\limits_{x \to 3} \dfrac{x^2}{x^2 - 4x + 3} = \dfrac{9}{9 - 4.3 + 3} = \dfrac{9}{0} = \infty \Rightarrow x = 3$ es a.v.

$\lim\limits_{x \to 3^-} \dfrac{x^2}{x^2-4x+3} \overset{x=2'9}{=} \dfrac{2'9^2}{2'9^2 - 4.2'9 + 3} \infty = \dfrac{+}{-} \infty = -\infty$

$\lim\limits_{x \to 3^+} \dfrac{x^2}{x^2-4x+3} \overset{x=3'1}{=} \dfrac{3'1^2}{3'1^2 - 4.3'1 + 3} \infty = \dfrac{+}{+} \infty = +\infty$

$x=3$

AH

$$\lim_{x\to-\infty} \frac{x^2}{x^2-4x+3} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1 \Rightarrow y=1 \text{ es a. h.}$$

En $-\infty$, $x=-1000 \rightarrow \dfrac{(-1000)^2}{(-1000)^2-4\cdot(-1000)+3} = \dfrac{1000000}{1004003} = 0'9\ldots$

En $+\infty$, $x=1000 \rightarrow \dfrac{1000^2}{1000^2-4\cdot 1000+3} = \dfrac{1000000}{996003} = 1'0\ldots$

Luego $y=1$

A.O

$\text{grd (núm)} = 2$

$-\ \text{grd (den)} = 2$

$0 \neq 1$ No hay A.O.

Ptos de corte

$x=0 \rightarrow y = \dfrac{0^2}{0^2-4\cdot0+3} = \dfrac{0}{3} = 0 \rightarrow (0,0)$

$y=0 \rightarrow \dfrac{x^2}{x^2-4x+3} = 0 \rightarrow x^2=0 \rightarrow x=0$

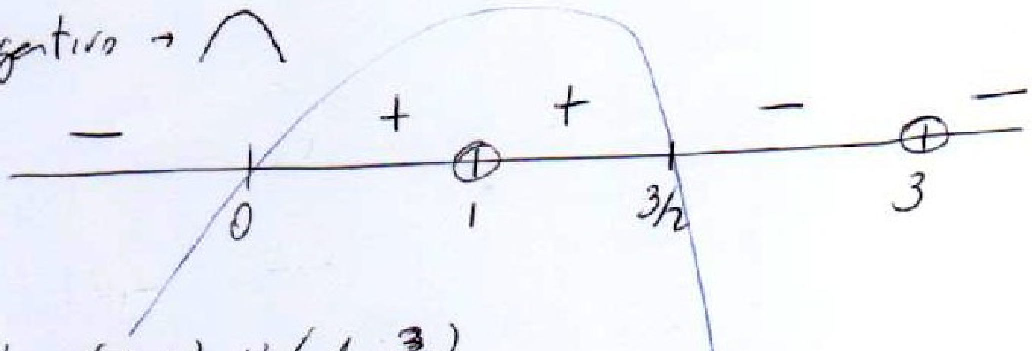
Monotonía

signo de y'

$-4x^2+6x=0 \Rightarrow x(-4x+6)=0 \begin{cases} x=0 \\ -4x+6=0 \rightarrow -4x=-6 \\ x=\dfrac{-6}{-4}=\dfrac{3}{2} \end{cases}$

$(x^2-4x+3)^2=0 \rightarrow x^2-4x+3=0 \rightarrow$ (resuelto en a)) $\begin{cases} x=1 \\ x=3 \end{cases}$

Como en y' el denominador está elevado al cuadrado, el signo de y' depende del numerador que es polinomio de 2º grado con coef. de x^2 negativo $\to \frown$

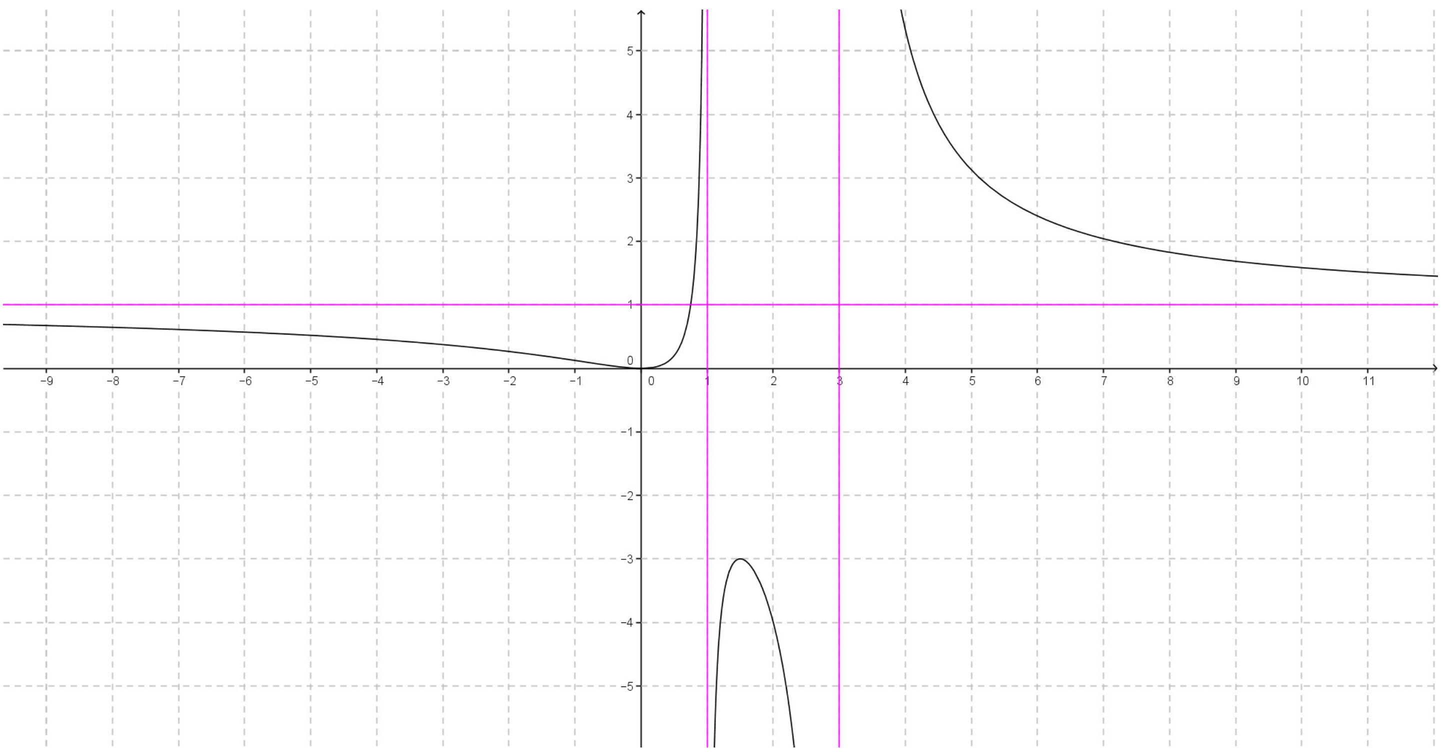


Creciente $(0,1) \cup (1, \frac{3}{2})$

Decreciente $(-\infty, 0) \cup (\frac{3}{2}, 3) \cup (3, +\infty)$

Máximo $x = \frac{3}{2} \to y = \frac{1'5^2}{1'5^2 - 4 \cdot 1'5 + 3} = -3$ Máximo $(\frac{3}{2}, -3)$

Mínimo $x = 0 \to$ (ya visto) $y = 0$ Mínimo $(0,0)$



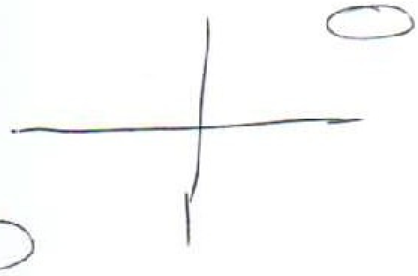
3) Representa $y = x^3 + 3x^2 - 9x + 5$

a) Dom $y = \mathbb{R}$

b) Ramas

$\lim\limits_{x \to -\infty} (x^3 + 3x^2 - 9x + 5) = \lim\limits_{x \to -\infty} x^3 = -\infty$

$\lim\limits_{x \to +\infty} (x^3 + 3x^2 - 9x + 5) = \lim\limits_{x \to +\infty} x^3 = +\infty$



c) Ptos de corte

$x = 0 \rightarrow y = 5 \rightarrow (0,5)$

$y = 0 \rightarrow x^3 + 3x^2 - 9x + 5 = 0$

	1	3	-9	5
1		1	4	5
	1	4	-5	0
1		1	5	
	1	5	0	
-5		-5		
	1	0		

$\rightarrow (1, 0)$ doble y $(-5, 0)$

d) $y' = 3x^2 + 6x - 9$ (parábola $\cup$)

e) Monotonía

signo de y'

$3x^2 + 6x - 9 = 0 \rightarrow x^2 + 2x - 3 = 0 \rightarrow x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4+12}}{2} =$

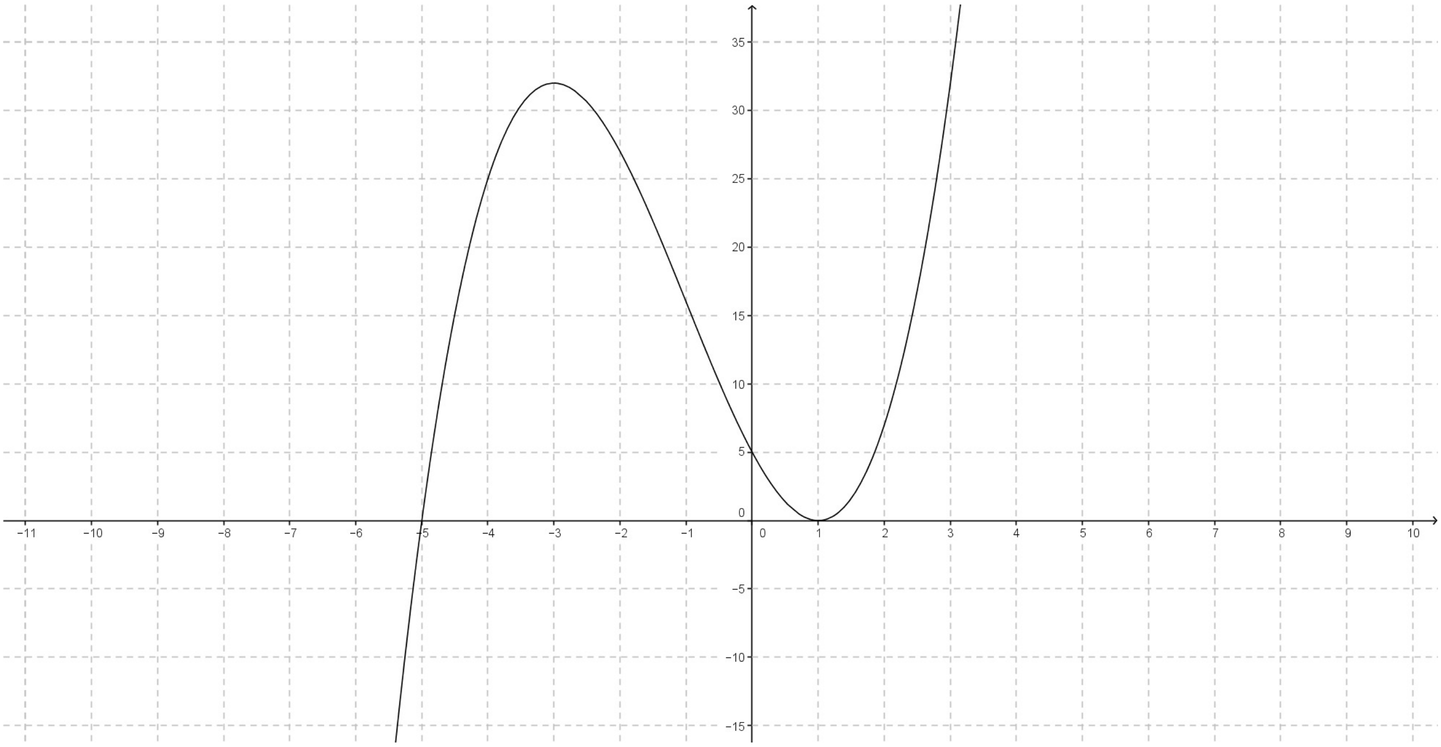
$= \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} = \begin{cases} 1 \\ -3 \end{cases}$

Creciente $(-\infty, -3) \cup (1, +\infty)$

Decreciente $(-3, 1)$

Máximo $x = -3$, $y = (-3)^3 + 3(-3)^2 - 9(-3) + 5 = 32 \rightarrow (-3, 32)$ Máx.

Mínimo $x = 1 \rightarrow (1, 0)$ Mínimo



35
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